

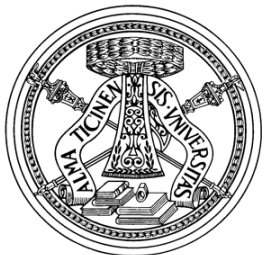
# Internal structure of the pion inspired by AdS/QCD correspondence

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## Part I – Ingredients

- Parton Distributions
- Light-Front Wave Functions (LFWFs)
- Tomography of the hadron



## Part II – Recipe

- AdS/QCD correspondence
- Pion LFWFs inspired by AdS/QCD

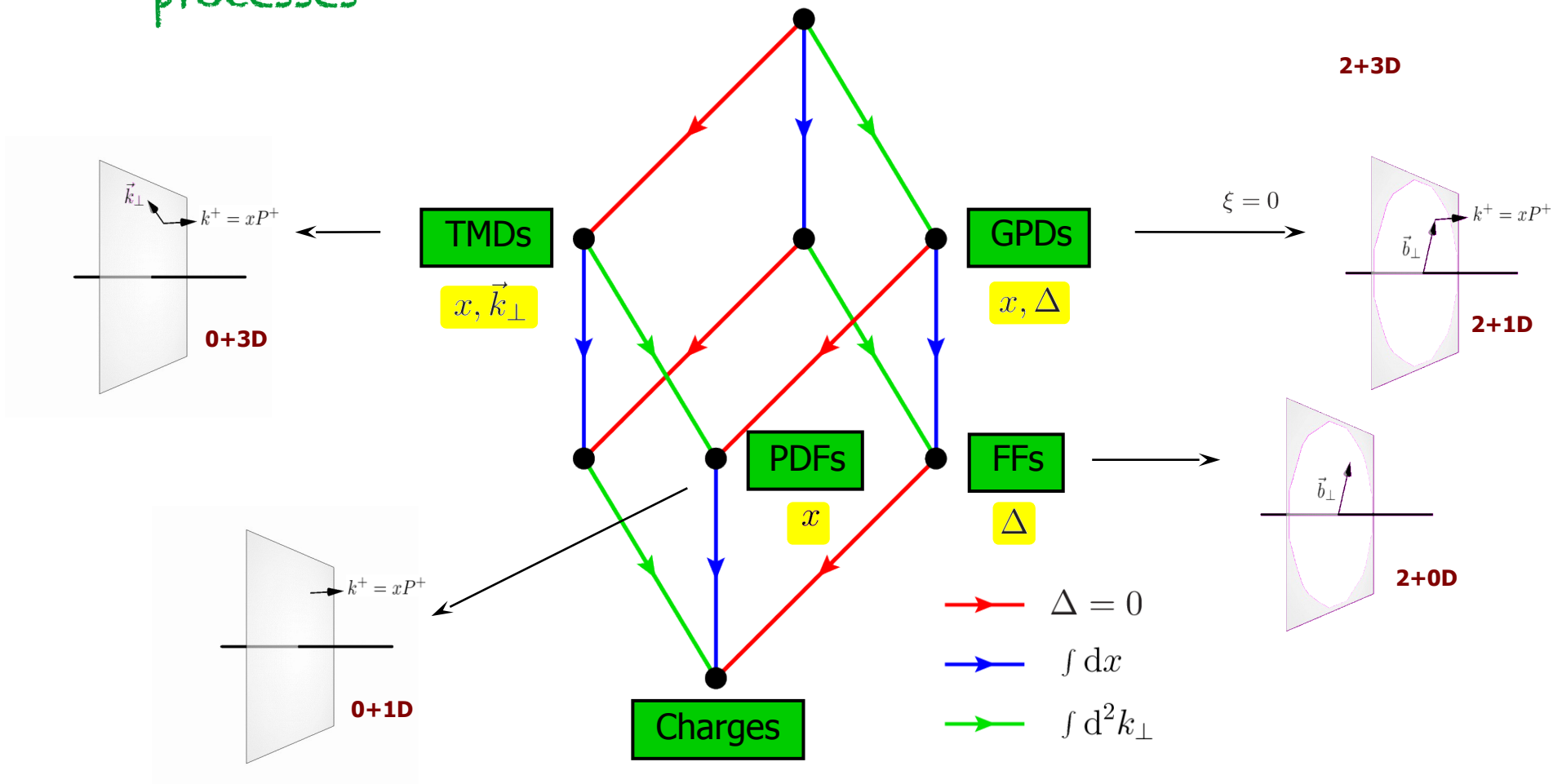


## Part III – Phenomenology

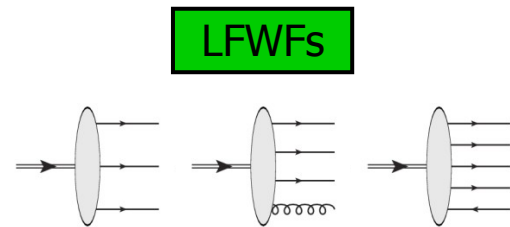
- Pion PDF and FF
- Results for the TMD and the strong coupling

## Conclusions





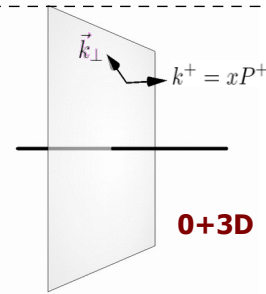
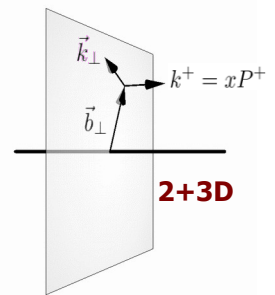
[ Lorcé, Pasquini, Vanderhaeghen (2011) ]



GTMDs

$x, \vec{k}_\perp, \Delta$

$\xi = 0$



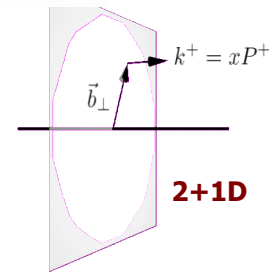
TMDs

$x, \vec{k}_\perp$

GPDs

$x, \Delta$

$\xi = 0$

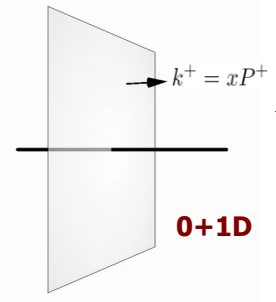
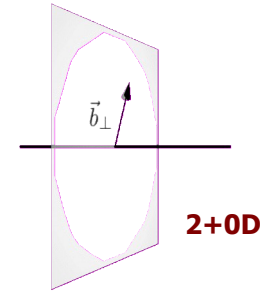


PDFs

$x$

FFs

$\Delta$

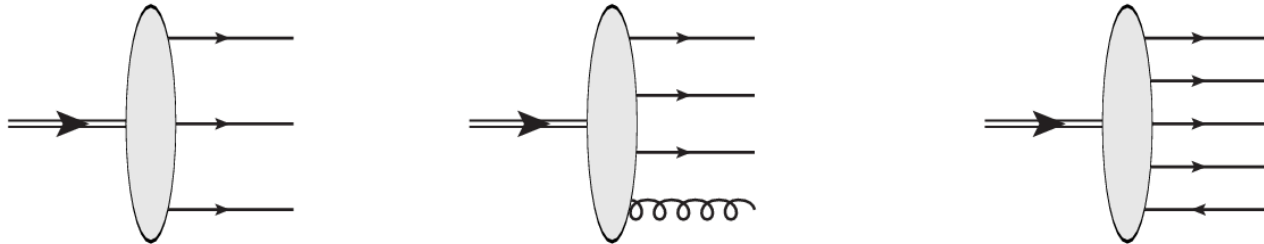


Charges

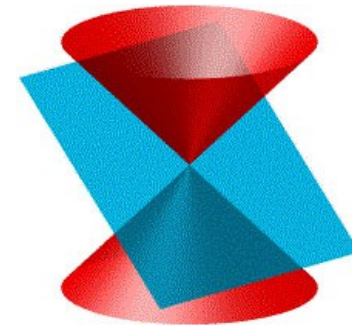
- $\Delta = 0$
- $\int dx$
- $\int d^2k_\perp$

## Fock expansion of the hadronic state (e.g. nucleon)

$$|p\rangle = \Psi_{qqq} |qqq\rangle + \Psi_{qqqg} |qqqg\rangle + \Psi_{qqqq\bar{q}} |qqqq\bar{q}\rangle + \dots$$



Convenient formalism → Light-Front quantization



$$x^+ = \frac{1}{\sqrt{2}} (x^0 + x^3); \quad x^- = \frac{1}{\sqrt{2}} (x^0 - x^3)$$

$$|P, \Lambda\rangle = \sum_{N, \beta} \int \left[ \frac{dx}{\sqrt{x}} \right]_N [d^2 k_{\perp}]_N \Psi_{N, \beta}^{\Lambda} (r_1, \dots, r_N) |N, \beta; \tilde{k}_1, \dots, \tilde{k}_N\rangle$$

**Light-front Wave Functions**

$\psi_N \rightarrow$  Probability amplitude to find the N-th state inside the proton

## Advantages:

- Positive longitudinal momenta
- Galilean subgroup of Poincaré group:  
non relativistic system in the transverse plane.
- LFWFs depend only on the intrinsic coordinates.
- All the hadronic observables are expressed in terms of overlap of LFWFs.
- When possible, the probabilistic interpretation of the parton distributions is evident.



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**PDF**

$$f_{1q}^{\Lambda}(x) = \frac{1}{2} \sum_{\beta} \int \frac{d^2 \mathbf{k}_{\perp}}{16\pi^3} |\psi_{\beta}^{\Lambda}(x, \mathbf{k}_{\perp})|^2$$

**EFF**

$$\mathcal{F}_{\Lambda\Lambda'}^q(Q^2) = 2P^+ \sum_{\beta=\beta'} e_q \int dx \int \frac{d^2 \mathbf{k}_{\perp}}{16\pi^3} \psi_{\beta'}^{*\Lambda'}(r') \psi_{\beta}^{\Lambda}(r)$$

**TMD**

$$f_{1q}^{\Lambda}(x, \mathbf{k}_{\perp}) = \frac{1}{2} \text{Tr}[\Phi_q] = \frac{1}{2} \frac{1}{16\pi^3} \sum_{\beta} |\psi_{\beta}^{\Lambda}(x, \mathbf{k}_{\perp})|^2$$

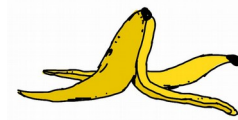


Are we ready to serve  
the hadron tomography?



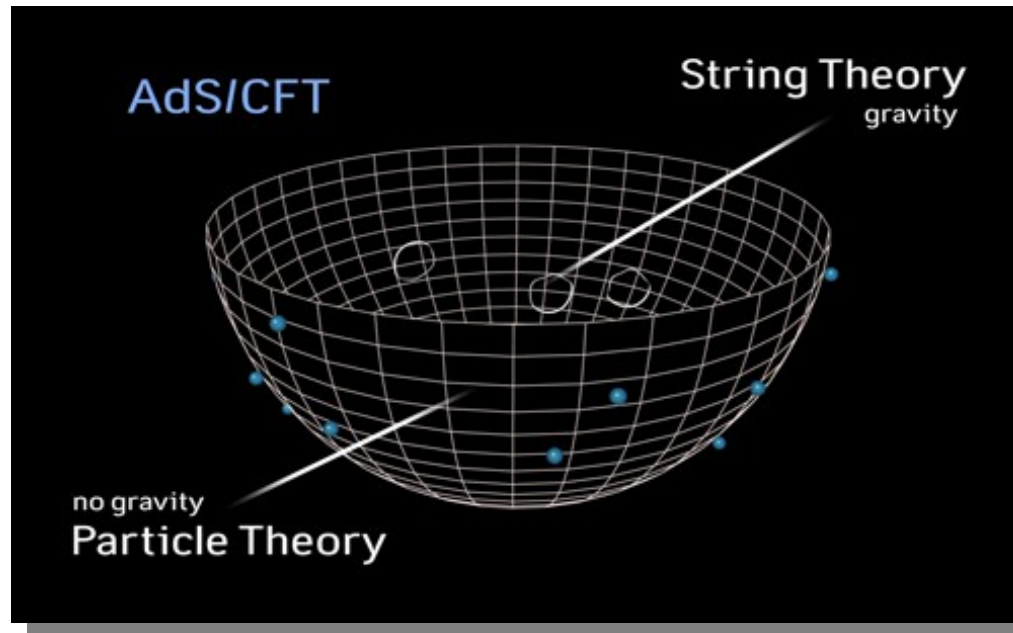


Are we ready to serve  
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**Problem:** LFWFs are non  
perturbative objects.  
How can we access them?

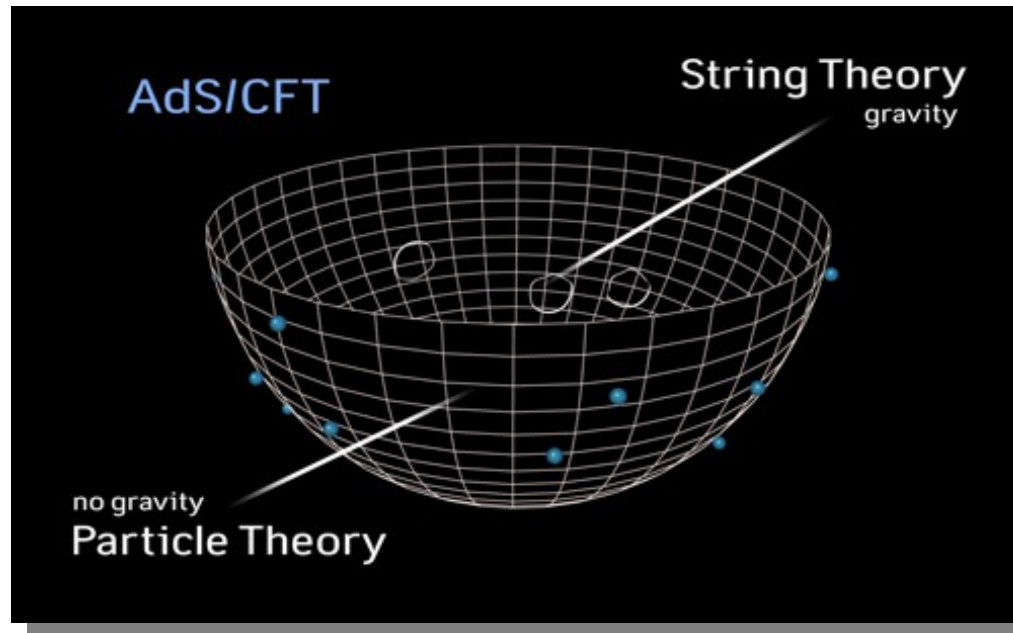
Possible approaches → Lattice QCD, Dyson-Schwinger Equation, **gauge/gravity duality**,...



$$ds^2 = \frac{R^2}{z^2} \left( \sum_{i=1}^3 dx_i dx^i - dz^2 \right)$$

Non gravitational dual field theory → Supersymmetric conformal Theory

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**Applicability to QCD?** → Massless quarks and non running coupling

$$\mathcal{L}_{QCD} = \bar{\psi}_i (i\gamma^\mu D_\mu)_{ij} \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

Bottom-up approach:

Modification of the AdS metric in order to obtain (a first approximation of ) QCD

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- Insertion of a dilaton field in order to have confinement:

$$S = \int d^4x dz \sqrt{|g|} e^{\varphi(z)} (g^{MN} \partial_M \Phi(x, z) \partial_N \Phi(x, z) - \mu^2 \Phi^2(x, z))$$

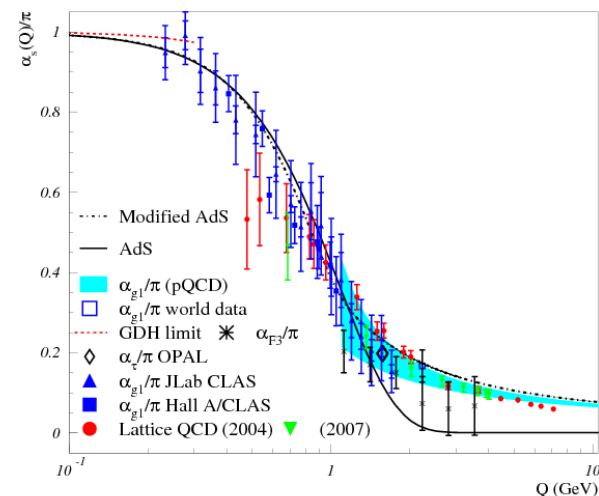
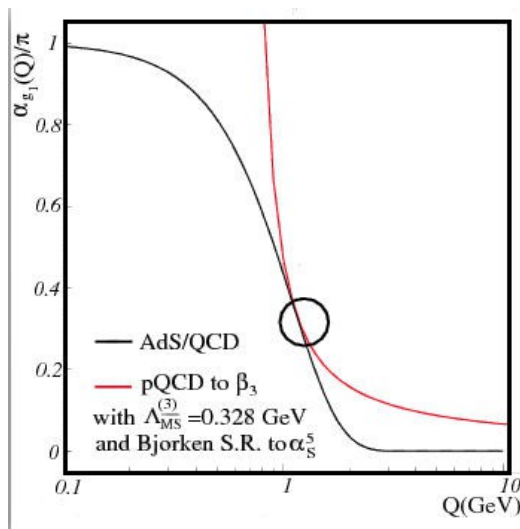
$$\varphi(z) = \kappa^2 z^2$$

Soft-wall model

$$e^{\varphi(z)} \rightarrow 1 \quad z \rightarrow 0$$

AdS<sub>5</sub>

- Freezing of the coupling constant at low energy (large distance):

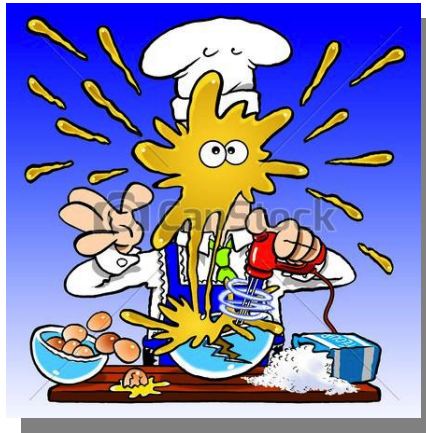


Matching the expression of the meson form factor:

$$\int d^4x \int dz \sqrt{g} A^M(x, z) \Phi_{P'}^*(x, z) \overleftrightarrow{\partial}_M \Phi(x, z)$$



$$(2\pi) \delta^{(4)}(P' - P - q) \epsilon_\mu (P' + P)^\mu F(q^2)$$

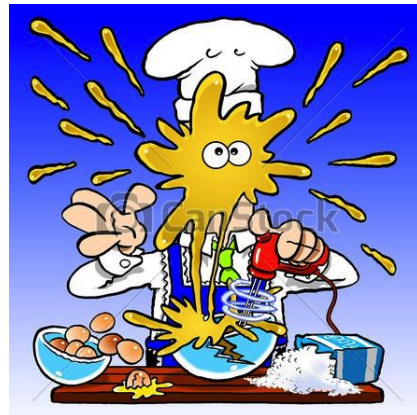


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Analytical expression for the valence state LFWF of the pion:

$$\psi_{q\bar{q}/\pi}(x, \mathbf{k}_\perp) = \frac{4\pi}{\kappa \sqrt{(1-x)x}} \sqrt{P_{q\bar{q}}} e^{-\frac{1}{2} \frac{\mathbf{k}_\perp^2}{\kappa^2 x(1-x)}}$$



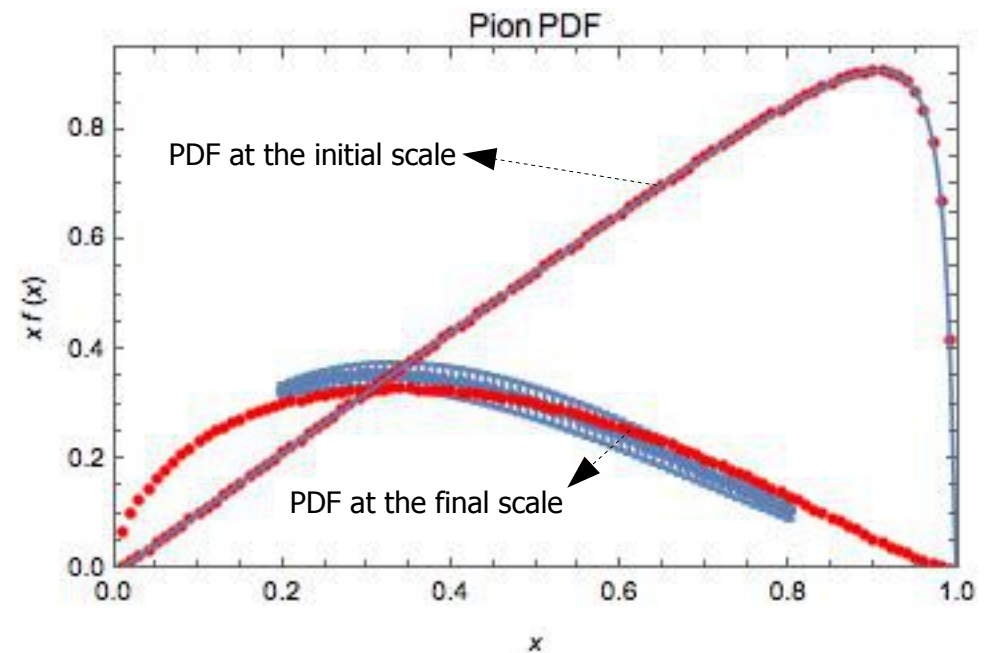
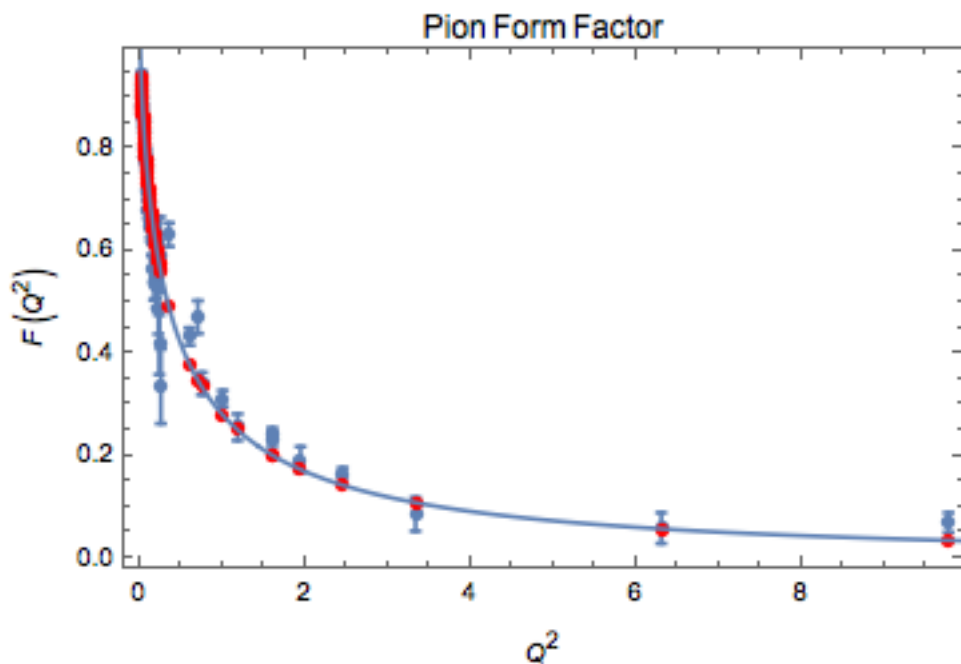
**Need for a phenomenological improvement!**



- We introduce the normalization constant and the quark mass parameters in a Lorentz invariant way (Brodsky's ansatz)
- We fit the free parameters using PDF parametrizations and FF experimental data
- We **obtain a set of parameters** which can be used for other **predictions** (e.g. TMD and running of the QCD coupling)

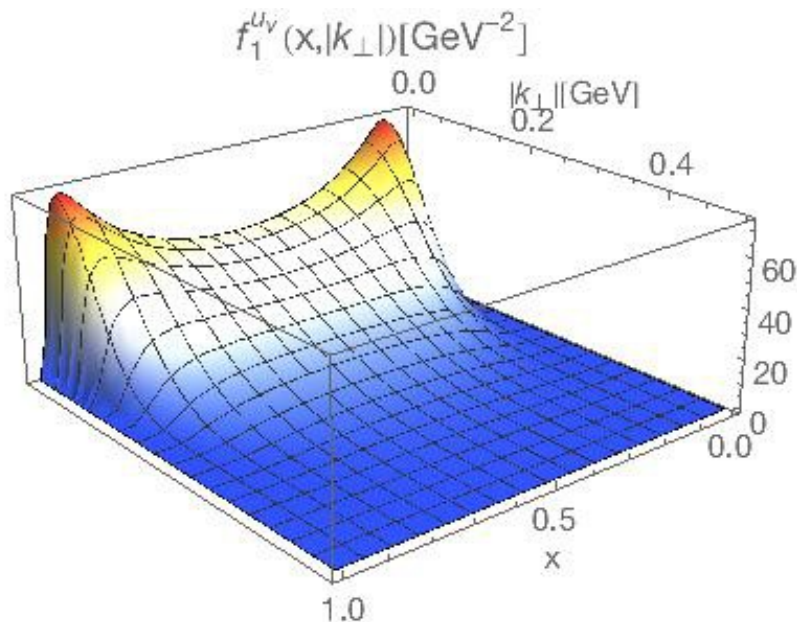
## Preliminary results

- Our fit
- Parametrization/experimental data



The PDF evolution is included in the fit procedure. The initial scale is a free parameter at this stage of the work.

# Preliminary results

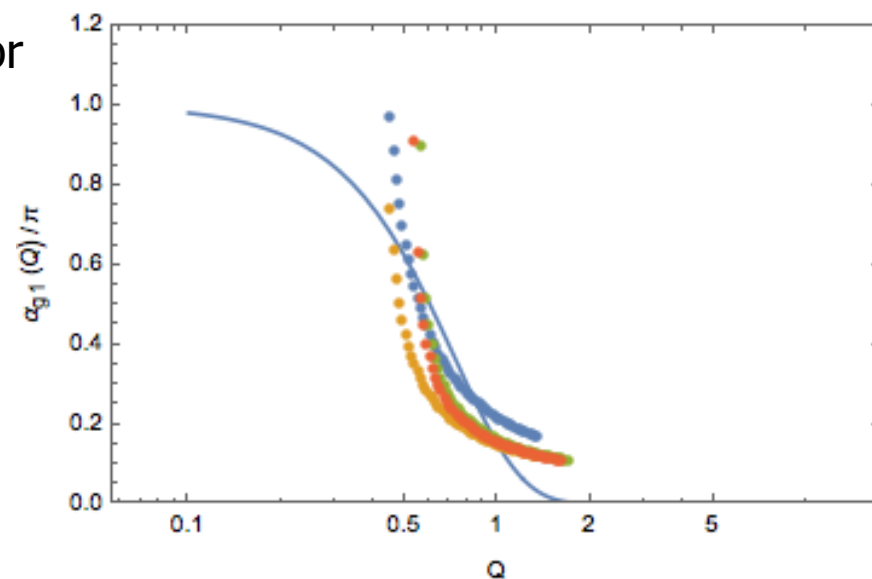


3D picture of the pion in momentum space: plot of the **unpolarized TMD** at the initial scale. QCD evolution is needed in order to compare this prediction with available (and future) experimental data.

## QCD running coupling

— Our prediction for the low energy behavior based on the AdS/QCD approach

- LO for  $\alpha_s(Q_0) = 0.139$
- NLO for  $\alpha_s(Q_0) = 0.119$
- NNLO for  $\alpha_s(Q_0) = 0.119$
- NNLO for  $\alpha_s(Q_0) = 0.120$



# Conclusions:

- Importance of the LFWFs to model parton distributions and get information on the internal structure of hadrons.
  - Need for a model which provides the hadronic LFWFs.
  - Meson LFWFs inspired by AdS/QCD correspondence.
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- Pure AdS/QCD correspondence provides LFWFs which have a rigid form.
  - No reasonable description of the pion.
  - Phenomenological changes are needed in order to describe the pion.
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  - Analysis of the running of the QCD coupling.

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**Thank you!**